

PROGRESSION FOR DEVELOPING ALGEBRA UNDERSTANDING THROUGH GENERALIZING ARITHMETIC ACROSS GRADES 3-7:

This curricular progression is intended to develop algebra understanding through generalizing arithmetic. Generalized arithmetic involves generalizing arithmetic relationships, including properties of number and operation, representing these relationships in diverse ways (e.g., using words, symbolic representations, or pictures), and justifying and reasoning explicitly with these generalizations. In this context, generalized arithmetic entails reasoning about the structure of arithmetic expressions rather than their computational value.

SUMMARY OF DEVELOPMENT OF IDEAS ACROSS GRADES 3-7:

The following summarizes a curricular progression for the development of algebra understanding generalizing arithmetic in grades 3-7. The progression is based on what we know from research that children can reasonably accomplish. It is not intended to be unique in its scope and sequence, but represents one possible progression for developing algebra understanding. In grades 3-5, students explicitly attend to generalizing, representing, and justifying arithmetic generalizations. This includes a strong emphasis on the fundamental properties of number and operation (e.g., Commutative Property of Addition). Other arithmetic generalizations, such as generalizations on classes of numbers (e.g., evens and odds) are also addressed. Representation-based arguments are introduced, and the limitations of case-based reasoning is explored. Students also begin to identify fundamental properties in order to justify computational work. Finally, students examine domains for which arithmetic generalizations are true.

Third-Grade:

Students begin generalizing Fundamental Properties of Number and Operation, focusing primarily on properties involving addition and subtraction. The order in which properties are introduced progresses from those that contain a single, repeated variable to those with multiple variables. (Simultaneous work with equations includes developing an understanding of equations in these various formats.) Fundamental properties based on multiplication are incorporated near the end of grade 3 after multiplication has been introduced. Students also begin to explore other arithmetic generalizations about classes of numbers and the operations, particularly those about sums of even numbers and odd numbers. Although generalizations in grade 3 are initially expressed in words, the use of variables to express arithmetic generalizations is also introduced. Students also begin to use Fundamental Properties to simplify computations and identify these properties in use. Finally, students begin to explore ways to justify generalizations, beginning with empirical arguments and moving towards general arguments based on reasoning with representations.

Core Actions:

- analyze information to develop a conjecture about the arithmetic relationship
- express the conjecture in words
- develop a justification or argument to support the conjecture's truth;
- explore different types of arguments, including empirical arguments, representation-based arguments, and arguments based on the algebraic use of number
- identify values for which the conjecture is true
- if appropriate, express the generalization using variables
- examine meaning of repeated variable in same equation
- for properties with multiple variables, examine the meaning of different variables in the same equation
- examine characteristic that generalization (property) is true for all values of the variable in a given number domain
- identify generalization (property) in use when doing computational work

Fourth-Grade Progression:

Students review previously established Fundamental Properties, including their symbolic forms. They continue generalizing new Fundamental Properties, focusing primarily on properties involving multiplication. All Fundamental Properties are examined within a grade-appropriate number domain. Students also continue to use Fundamental Properties to simplify computations and identify these properties in use. They continue their work with other arithmetic generalizations involving classes of numbers (e.g. evens and odds). In particular, they extend their understanding of generalizations about sums of even numbers and odd numbers to include generalizations about products. Additionally, they examine generalizations that incorporate previously established generalizations (e.g., ' $a + b - b = a$ ' incorporates the Fundamental Properties, $b - b = 0$ and $a + 0 = a$, addressed in Grade 3). They also continue to develop their understanding of types of arguments used to justify generalizations, extending these forms from empirical arguments and representation-based arguments to arguments based on reasoning with previously established generalizations. Finally, in order to begin establishing the limitations of empirical arguments used to justify arithmetic generalizations, they begin to compare and contrast the strengths of empirical arguments with more general arguments.

Core Actions:

- analyze information to develop a conjecture about the arithmetic relationship
- express the conjecture in words and, if appropriate, variables
- develop a justification using, as appropriate, an empirical argument, algebraic use of number argument, representation-based argument, or argument based on reasoning with previously established
- examine the limitations of an empirical argument
- identify number domain on which conjecture is true, including extending number domains for which generalizations were previously established to examine whether generalization still holds
- examine meaning of repeated variables in same equation
- examine meaning of different variables in same equation
- examine constraints on values of variable (i.e., a cannot be 0 to avoid division by 0)
- examine characteristic that generalization (property) is true for all values of the variable in a given number domain
- identify generalization/property in use when doing computational work

Fifth-Grade Progression:

Students review previously established Fundamental Properties, including their symbolic forms. Fundamental Properties are re-examined in terms of their validity on number domains broader than those previously used. In particular, students examine whether Fundamental Properties are valid for fractions. They also continue to use Fundamental Properties to simplify computations and identify these properties in use. They continue to explore other arithmetic generalizations, extending their work with generalizations about classes of numbers (e.g., evens and odds) to include generalizations about outcomes of calculations (e.g., Does multiplying two numbers always produce a larger result?). Finally they continue to develop their understanding of different types of arguments to support generalizations and compare and contrast the strength of more general arguments with empirical arguments.

Core Actions:

- analyze information to develop a conjecture about the arithmetic relationship
- express the conjecture in words and, if appropriate, variables
- develop a justification using, as appropriate, an empirical argument, algebraic use of number argument, representation-based argument, or argument based on reasoning with previously established
- examine the limitations of an empirical argument
- identify number domain on which conjecture is true, including extending number domains for which generalizations were previously established to examine whether generalization still holds
- examine meaning of repeated variables in same equation
- examine meaning of different variables in same equation
- examine any constraints on values of variable (i.e., a cannot be 0 to avoid division by 0)
- express generalizations (equations) in different equivalent forms (e.g., $a = b + c$ is equivalent to $a - c = b$)
- examine characteristic that generalization (property) is true for all values of the variable in a given number domain
- identify generalization/property in use when doing computational work

Sixth-Grade Progression:

Students review previously established Fundamental Properties, including their symbolic forms. Fundamental Properties are re-examined in terms of their validity on number domains broader than those previously used. In particular, students examine whether Fundamental Properties are valid for decimals. Students extend their work in identifying Fundamental Properties as they are used in computations to include identifying their use in transformations on simple algebraic expressions. They also continue to explore arithmetic generalizations, including generalizations about classes of numbers and outcomes of calculations. Finally they continue to develop their understanding of different types of arguments to support generalizations and compare and contrast the strength of more general arguments with empirical arguments.

Seventh-Grade Progression:

Students review previously established Fundamental Properties, including their symbolic forms. Fundamental Properties are re-examined in terms of their validity on number domains broader than those previously used. In particular, students examine whether Fundamental Properties are valid for integers. Students continue their work on developing arithmetic generalizations and identifying the Fundamental Properties in use in transforming simple algebraic expressions. They also continue to develop strategies for justifying generalizations by examining symbolic arguments for simple sums of even numbers and odd numbers. They compare the strength of symbolic arguments to other types, such as empirical arguments or representation-based arguments, appreciating the limitations of empirical arguments and the need for more general ones.