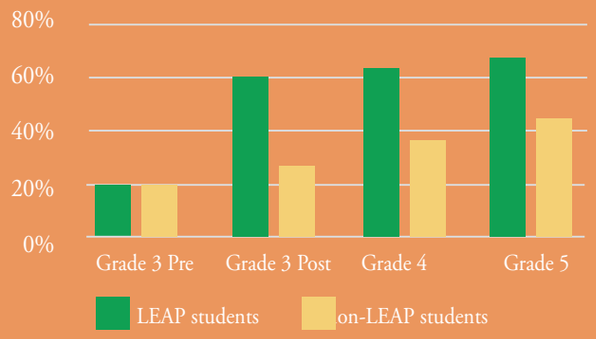
WHY EARLY ALGEBRA?

Algebra continues to serve as a gatekeeper for many students, limiting their prospects for future educational and employment opportunities. The "arithmetic-then-algebra" approach, where an arithmetic curriculum in the elementary grades is followed by a formal treatment of algebra in the secondary grades, has not enabled students to successfully navigate the transition from the concrete, arithmetic reasoning of elementary school mathematics to the increasingly complex, abstract algebraic reasoning required for secondary school mathematics and beyond.

Over the course of the past decade, Project LEAP has developed research-based and classroom-tested early algebra curricular materials designed to help elementary school students develop an understanding of foundational algebraic concepts and skills so that they are better prepared for success in secondary school algebra.

As the graph displays, Project LEAP students significantly outperformed their peers (who experienced typical elementary school lessons) on challenging assessment items designed to measure their understanding of fundamental algebraic concepts and skills. As a result of their early algebra experience, Project LEAP students will enter middle school with a stronger foundation for successfully learning algebraic concepts and skills.

OVERALL CORRECTNESS



TO LEARN MORE:

To learn more about the research behind Project LEAP, visit our web site: http://algebra.wceruw.org

THE UNIVERSITY of WISCONSIN MADISON

The City College of New York

ABOUT LEAP PROFESSIONAL DEVELOPMENT

Teachers who participate in LEAP professional development will:

- enhance their understanding of early algebra concepts and practices
- develop their algebra "eyes and ears"
  - learn to design tasks to "algebrafy" their curriculum
  - learn to recognize and capitalize on classroom opportunities to engage students in algebraic thinking
- learn to facilitate meaningful mathematical conversations
- learn to help their students develop a foundation for successfully learning algebraic concepts and skills in middle school
- deepen their understanding of Common Core Standards for Mathematical Practice
- learn how to strengthen students' understanding of arithmetic to support computational skills

PROJECT LEAP TEAM

Dr. Maria Blanton

Maria Blanton is a Senior Scientist at TERC in Cambridge, MA. Her primary research interest is teaching and learning algebra in the elementary grades.

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Ana Stephens is an Associate Researcher at the Wisconsin Center for Education Research at the University of Wisconsin-Madison. Her research interests concern the development of students' and teachers' algebraic reasoning, particularly in the elementary grades.

Angela Murphy Gardiner

Angela Murphy Gardiner is a Senior Research Associate at TERC. Prior to joining TERC she was an elementary school teacher in Massachusetts. Her interests focus on early algebra and working with teachers.

TO LEARN MORE:

To learn more about Project LEAP, its early algebra curricular materials, and LEAP Professional Development, contact: angela_gardiner@terc.edu, 617-873-9674

TERC

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PROJECT LEAP

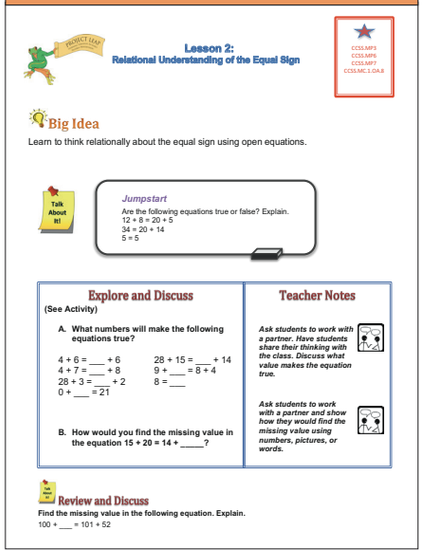
Learning Through an Early Algebra Progression

Research-based and classroom-tested early algebra curricular materials and professional development

ABOUT THE LEAP CURRICULAR MATERIALS

The research-based and classroom-tested curricular materials consist of approximately 20 early algebra lessons per grade level — teacher lesson plans, student activity sheets, and expanded lesson materials (see sample lesson and expanded lesson overview).

SAMPLE LESSON



PROJECT LEAP

Lesson 2:
Relational Understanding of the Equal Sign

CCSS.MP3
CCSS.MP6
CCSS.MP7
CCSS.MC.1.OA.8

Big Idea

Learn to think relationally about the equal sign using open equations.

Talk About It!

Jumpstart

Are the following equations true or false? Explain.
12 + 8 = 20 + 5
34 = 20 + 14
5 = 5

Explore and Discuss

(See Activity)

A. What numbers will make the following equations true?

4 + 6 = ___ + 6
4 + 7 = ___ + 8
28 + 3 = ___ + 2
0 + ___ = 21
28 + 15 = ___ + 14
9 + ___ = 8 + 4
8 = ___

B. How would you find the missing value in the equation 15 + 20 = 14 + _____?

Teacher Notes

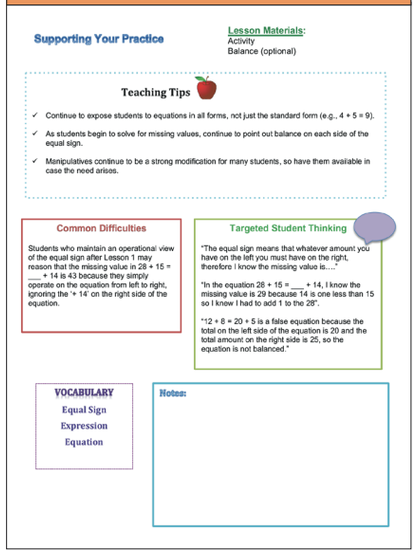
Ask students to work with a partner. Have students share their thinking with the class. Discuss what value makes the equation true.

Ask students to work with a partner and show how they would find the missing value using numbers, pictures, or words.

Review and Discuss

Find the missing value in the following equation. Explain.
100 + ___ = 101 + 52

SAMPLE LESSON



Supporting Your Practice

Lesson Materials:
Activity
Balance (optional)

Teaching Tips

- ✓ Continue to expose students to equations in all forms, not just the standard form (e.g., 4 + 5 = 9).
- ✓ As students begin to solve for missing values, continue to point out balance on each side of the equal sign.
- ✓ Manipulatives continue to be a strong modification for many students, so have them available in case the need arises.

Common Difficulties

Students who maintain an operational view of the equal sign after Lesson 1 may reason that the missing value in 28 + 15 = ___ + 14 is 43 because they simply operate on the equation from left to right, ignoring the '+ 14' on the right side of the equation.

Targeted Student Thinking

"The equal sign means that whatever amount you have on the left you must have on the right, therefore I know the missing value is…."

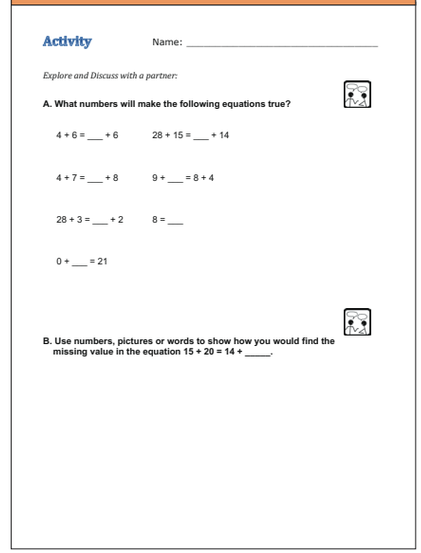
"In the equation 28 + 15 = ___ + 14, I know the missing value is 29 because 14 is one less than 15 so I know I had to add 1 to the 28".

"12 + 8 = 20 + 5 is a false equation because the total on the left side of the equation is 20 and the total amount on the right side is 25, so the equation is not balanced."

VOCABULARY
Equal Sign
Expression
Equation

Notes:

SAMPLE LESSON



Activity Name: _____

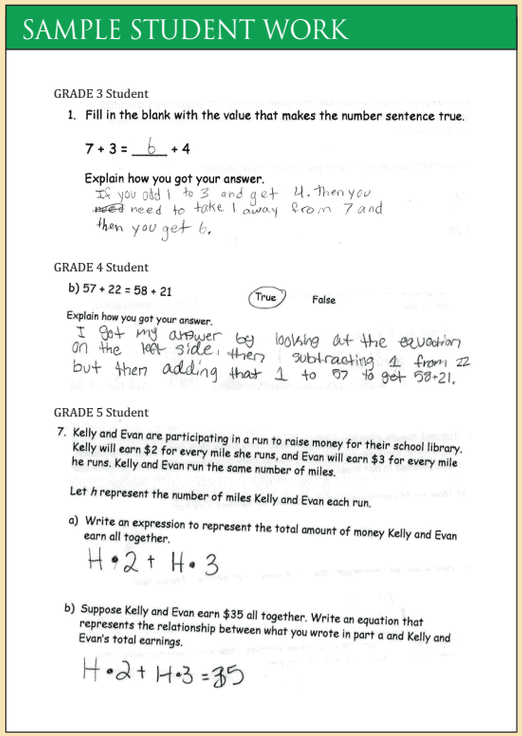
Explore and Discuss with a partner:

A. What numbers will make the following equations true?

4 + 6 = ___ + 6
4 + 7 = ___ + 8
28 + 3 = ___ + 2
0 + ___ = 21
28 + 15 = ___ + 14
9 + ___ = 8 + 4
8 = ___

B. Use numbers, pictures or words to show how you would find the missing value in the equation 15 + 20 = 14 + _____.

SAMPLE STUDENT WORK



GRADE 3 Student

1. Fill in the blank with the value that makes the number sentence true.

7 + 3 = __6__ + 4

Explain how you got your answer.
If you add 1 to 3 and get 4. then you need to take 1 away from 7 and then you get 6.

GRADE 4 Student

b) 57 + 22 = 58 + 21 True False

Explain how you got your answer.
I got my answer by looking at the equation on the left side, then subtracting 1 from 22 but then adding that 1 to 57 to get 58+21.

GRADE 5 Student

7. Kelly and Evan are participating in a run to raise money for their school library. Kelly will earn $2 for every mile she runs, and Evan will earn $3 for every mile he runs. Kelly and Evan run the same number of miles.

Let *h* represent the number of miles Kelly and Evan each run.

a) Write an expression to represent the total amount of money Kelly and Evan earn all together.

H•2 + H•3

b) Suppose Kelly and Evan earn $35 all together. Write an equation that represents the relationship between what you wrote in part a and Kelly and Evan's total earnings.

H•2 + H•3 = 35

EXPANDED LESSON OVERVIEW

Each lesson includes "expanded" lesson materials that provide additional information about the lesson, and includes the following resources:

LESSON OBJECTIVES are provided at the beginning of each lesson plan. These describe goals for students' activity and understanding.

JUMPSTARTS are problems posed to students at the beginning of each lesson. They provide an opportunity to revisit important concepts throughout the year. Ask students to think about each jumpstart on their own or in small groups and then discuss as a whole group, eliciting responses and strategies from students. Encourage students to respond to each other's contributions.

The **BIG IDEAS** that are addressed in each lesson are described in Lesson Connections. You should have these in mind throughout your teaching of the lesson so that your questions, your organization of student discussion, and your examination of student thinking are all centered on the mathematical concepts of most importance.

ABOUT STUDENT RESPONSES helps you anticipate how students might respond to the tasks in each lesson and how you might respond in turn. These expected responses are based on what we know from research on children's algebraic thinking.

TEACHING SUGGESTIONS to keep in mind throughout each lesson offer guidance as you implement the tasks in the lessons. Suggestions are related to how research tells us students are likely to respond to the lesson's tasks and the algebraic ideas we want students to understand.

RATIONALE FOR THE TASKS included in each lesson are provided to explain why particular tasks were chosen and describe what they can reveal about students' thinking.

MATHEMATICAL CONVENTIONS relevant to each lesson are noted to ensure symbols and mathematical notations are used in ways consistent with accepted practices.

COMMON CORE Standards for Mathematical Content and Standards for Mathematical Practice that are addressed are listed at the end of each lesson.

TEACHER TESTIMONIALS

"Well to be honest, the lessons made me realize my students can do algebra and they can be successful at it…. I didn't think 3rd and 4th graders were capable before, so I kind of changed my mind. I expect them to do it now." (4th grade teacher)

"It's amazing how they can reason and solve problems in so many different ways that you just don't expect them to at 4th grade level." (4th grade teacher)

"I think they are getting more in-depth experience with algebraic thinking, more in depth with equations, properties, reasoning skills, and communication skills because they have to share so much of their thinking." (3rd grade teacher)

"I think it's a really good program and hopefully it will get adopted by more schools." (3rd grade teacher)