PROGRESSION FOR DEVELOPING ALGEBRA UNDERSTANDING FOR EQUALITY, EXPRESSIONS, EQUATIONS, AND INEQUALITIES

This curricular progression is intended to develop algebra understanding through a study of the equal sign, expressions, equations, and inequalities. An equation is a mathematical statement indicating the equivalence of two quantities or expressions. An inequality is a mathematical statement indicating the relative magnitudes of two quantities or expressions that are not equivalent.

SUMMARY OF DEVELOPMENT OF IDEAS ACROSS GRADES 3-7:

The following summarizes a proposed curricular progression for the development of algebra understanding through the study of the equal sign, expressions, equations and inequalities in grades 3-8. The progression is based on what we know from research that children can reasonably accomplish. It is not intended to be unique in its scope and sequence, but represents one possible progression for developing algebra understanding. Students explicitly attend to developing a relational understanding of the equal sign in grades 3-5 (although this is not explicitly addressed after grade 5). Students in grades 3-5 begin preliminary work on solving simple linear equations, where the work focuses on analyzing the structure of equations to develop insightful solutions rather than routinely applying procedures. Students explore Properties of Equality and develop understanding of Properties of Equations in grades 3-5. Algebraic expressions, equations and inequalities are used to model problem situations and reason quantitatively about relative magnitudes, expressing the relationships using algebraic notation, as a first step in understanding inequality relationships.

Third Grade:

In 3rd grade, students begin developing a relational understanding of the equal sign. To support this, they critically examine the meaning of the symbol '=' through the exploration of true/false and open equations (that is, missing value problems). True false and open equations are initially posed in terms of addition tasks. These are later extended to include the operations of subtraction and multiplication. As students develop a relational understanding of the equal sign, they are beginning informal work on solving equations and Properties of Equations. They extend their understanding by modeling linear problem situations using algebraic expressions and equations. They solve simple linear equations by analyzing the structure of the equation for insights into a solution rather than focusing on procedures for solving equations. Solving equations not only reinforces the development of a relational understanding of '=', it also supports the development of an informal understanding of Properties of Equality (e.g., Symmetric Property) by expressing equations in different formats. Finally, students begin to develop their understanding of inequalities by comparing numerical and algebraic quantities and using inequalities to express their relative magnitudes.

Types of Equations: single variable one-step or two-step linear equations of the form $x + a = b$, $ax = b$, or $ax + b = c$
Expressions and Inequalities are in linear forms comparable to equations

Core Actions:

Equality

- identify different meanings of '=', including as expressing a relationship between quantities
- interpret equations (number sentences) written in various formats (e.g., other than $a + b = c$)
- solve missing value problems by interpreting the equal sign relationally and reasoning from the structural relationship in the equation

Expressions

- identify variable(s) to represent an unknown quantity or quantities in a problem situation
- describe a quantity as an algebraic expression using variables
- interpret an algebraic expression in the context of a problem
- identify different (equivalent) ways to write an expression

Equations

- model problem situations to produce linear equations of the form x + a = b, ax = b, and ax + b =c.
- identify variable(s) to represent an unknown quantity in a linear problem situation
- represent equivalent algebraic expressions in an equation
- represent the relationship of two equivalent expressions as an equation
- identify different (equivalent) ways to write an equation (i.e., a = b or b = a)
- analyze the structure of an equation to determine the value of a variable.
- check the solution by substituting the value of the variable in the original equation, or determine if the solution is reasonable given the context of the problem
- examine role of variable as an unknown fixed or unknown varying quantity

Inequalities

- identify variables to represent two unspecified (unmeasured) quantities of different amounts
- examine meaning of different variables in same inequality
- represent the inequality relationship between two quantities or algebraic expressions
- identify all possible ways to express an inequality relationship

Fourth Grade:

In 4th grade, students continue to develop a relational understanding of the equal sign by solving equations and interpreting '=' in tasks that involve addition, subtraction, multiplication, or division. They continue to model problem situations using algebraic expressions and equations. They continue to develop an informal understanding of Properties of Equality, focusing on the symmetric and transitive properties. They also extend work on solving equations to include solving equations with a repeated variable. The emphasis in solving equations is on analyzing the structure of the equation rather than on applying routine procedures for solving equations. Finally, students continue to develop their understanding of inequalities by comparing algebraic quantities and using inequalities to express their relative magnitudes.

Types of Equations: single variable one-step or two-step linear equations of the form $x + a = b$, $ax = b$, or $ax + b = c$, two-step linear equations with repeated variables, linear equations in two variables

Expressions and Inequalities are in linear forms comparable to equations.

Core Actions:

Equality

- review different meanings of '=', including as expressing a relationship between quantities
- interpret equations (number sentences) written in various formats (e.g., other than $a + b = c$)
- solve missing value problems by interpreting the equal sign relationally and reasoning from the structural relationship in the equation

(Develop informal understanding of Symmetric Property of Equality)

- develop an equation that expresses the relationship between two unspecified quantities of equal amounts
- identify all possible ways to express the relationship
- describe generalization informally (that if $a = b$, then $b = a$)

Expressions

- identify variable(s) to represent the unknown quantity or quantities in a problem situation
- represent the quantity as an algebraic expression using variables
- interpret an algebraic expression in the context of the problem
- explore why expressions are different than equations (i.e., expressions are not to be 'solved')

Equations

- model problem situations to produce a linear equations* of the form ax = b or linear equations with a variable repeated (e.g., ax + x = b).
- model problem situations to produce linear equations in two variables
- identify variable(s) to represent the unknown quantity or quantities in a problem situation
- represent the relationship of two equivalent expressions as an equation
- for linear equations with repeated variables, examine the meaning of a repeated variable in the same equation
- analyze the structure of the equation to determine the value of variable.
- check the solution of an equation by substituting the value of the variable in the original equation or determine if the solution is reasonable given the context of the problem
- informally examine role of variable as a fixed or varying unknown

Inequalities

- identify variables to represent two unspecified (unmeasured) quantities of different amounts
- examine meaning of different variables in same inequality
- represent the inequality relationship between two quantities or algebraic expressions
- identify all possible ways to express an inequality relationship

Fifth Grade:

In 5th grade, students review their relational understanding of the equal sign by solving and interpreting tasks that use inverse operations (i.e., a task might include both multiplication and division or both addition and subtraction). They continue to model linear problem situations using algebraic expressions and equations and extend their work in transforming algebraic expressions to include applying the Distributive Property to expand or simplify algebraic expressions. They extend their understanding of identifying the Fundamental Properties in computational work to identifying properties used when transforming algebraic expressions. They strengthen their understanding of solving simple equations, and extend this to more complex equations. As in previous grades, the emphasis on solving equations is on analyzing the structure of an equation as a way to determine a solution rather than on applying formal procedures. As a precursor to the study of formal procedures for solving equations in middle grades, students explore and generalize Properties of Equations for addition and subtraction (i.e., if a = b, then a + c = b + c; if a = b, then a – c = b – c). Finally, students strengthen their understanding of inequalities by comparing algebraic expressions and using inequalities to express their relative magnitudes.

Types of Equations: single variable one-step or two-step linear equations of the form x + a = b, ax = b, or ax + b = c; linear equations with repeated variables (e.g., ax + bx = c); linear equations in two variables

Inequalities: linear forms comparable to those used in equations and expressions

Core Actions:

Equality

- solve and interpret equations (number sentences) written in various formats (e.g., other than a + b = c)
- identify solutions to missing value problems by interpreting the equal sign relationally and reasoning from the structural relationship in the equation

Expressions

- apply Fundamental Properties to transform simple algebraic expressions of the form ax + bx and ax – bx and (a+b)x or (a-b)x into the appropriate forms.

- explore why expressions are different than equations (i.e., expressions are not to be 'solved')
- identify variable(s) to represent the unknown quantity or quantities in a problem situation
- represent the quantity as an algebraic expression using variables
- interpret an algebraic expression in the context of the problem
- explore why expressions are different than equations (i.e., expressions are not to be 'solved')

Equations

- model problem situations using (linear) equations in one or two variables (e.g., ax + b = c) or repeated variables (e.g. ax + bx = c)
- identify variable(s) to represent an unknown quantity or quantities
- represent the relationship of two equivalent expressions as an equation
- solve two-step, three-step and repeated variable linear equations by examining the structure of the equation (e.g., 3x + 2x = 15)
- examine the meaning of a repeated variable in the same equation
- check the solution of an equation by substituting the value of the variable in original equation or determine if the solution is reasonable given the context of the problem
- informally examine the role of variable as fixed or varying unknown

(Properties of Equations)

- analyze computations to develop a generalization about the Addition Property of Equations (If a = b, then a + c = b + c or "adding the same quantity to both sides of an equation preserves the relationship") and the Subtraction Property of Equations (If a = b, then a - c = b – c, or "subtracting the same quantity from both sides of an equation preserves the relationship")
- express generalizations about properties of equations in words
- identify number domain on which the generalizations are true
- express the generalizations (properties) using variables
- identify generalizations (properties) in use in computations

Inequalities

- identify variables to represent two unspecified (unmeasured) quantities of different amounts
- examine meaning of different variables in the same inequality
- represent the inequality relationship between two quantities or algebraic expressions
- identify all possible ways to express the relationship

Sixth Grade:

In 6th grade, students continue to model linear problem situations using algebraic expressions and equations. They extend their study of Properties of Equations by exploring and generalizing these properties for the operations of multiplication and division (i.e., if $a = b$, then $a \times c = b \times c$; if $a = b$ and $c \neq 0$, then $a \div c = b \div c$). They also continue their work on transforming linear algebraic expressions and solving simple linear equations and extend this to include solving multi-step problems with repeated variables. Although the focus in solving equations is on seeing the structure in an equation rather than routinely applying procedures, they begin to informally identify properties of equations in use as they solve equations. They continue to identify the Fundamental Properties used in transforming algebraic expressions. They develop a more formal understanding of the Properties of Equality by generalizing the Symmetric Property and Transitive Property. Finally, students strengthen their understanding of inequalities by comparing algebraic expressions and using inequalities to express their relative magnitudes.

Seventh Grade:

In 7th grade, students continue to model linear problem situations using algebraic expressions and equations. They also continue to develop their understanding of transforming linear algebraic expressions and solving linear equations, including identifying the Fundamental Properties used in transforming algebraic expressions. They extend their understanding of solving equations to include a more formal use of Properties of Equations in solving equations and identifying these properties in use. Finally, they extend their understanding of inequalities by solving linear inequalities graphically and exploring whether Properties of Equality hold for inequalities.