VARIABLE

Variables are linguistic tools that allow us to express mathematical ideas in succinct ways through symbolic notation. Note that in this progression variables refer only to letters (literal symbols).

STRAND 2. Representations (Representing Generalizations):

Variables are an essential tool for representing generalizations. This strand addresses the different meanings and interpretations of variable.

CONSTRUCT (This column contains constructs within the Representations strand.)	**CLAIMS (UNDERSTANDINGS)** (This column contains understandings that students are expected to eventually acquire over the grades 3-8 progression. They will have partial understandings and misconceptions along the way towards these understandings.)	**EVIDENCE** (This column contains what students' oral or written work or activity should include.)	**DIFFICULTIES & MISCONCEPTIONS** (This column contains common misconceptions or difficulties revealed by research.)	**RESEARCH BASE** (This column contains what research tells us students can do.)	**TASKS** (This column contains research-based tasks.)
2.1) A variable represents the measure or amount of the object, not the object itself.	• Understand that a variable represents the measure or amount of the object, not the object itself. • Understand that labels (such as m for meters) serve as mnemonic devices, not as variables.	• Descriptions of variables as representing measures or amounts of a fixed but unknown or varying quantity (e.g., n represents "number of handshakes", not "handshakes") • Appropriate description of the meaning of the variable	• Students confuse the variable as representing the object or quantity being measured rather than the magnitude of the quantity itself. (Booth, 1988; Clement, 1982; Rosnick, 1981; Stacey & MacGregor, 1997; grades 7-10) • Students are more apt to view letters as labels when the symbols are mnemonic (McNeil		• Candy Problem (Carraher et al., 2006, grades 2-4) • Literal symbol interpretation (Knuth et al., 2005, grades 6-8) • Cakes and brownies (McNeil et al., 2010, grades 6-8)

			et al., 2010) (6-8) • Students may believe letters are general referents (e.g., h stands for height of multiple people) (MacGregor & Stacey, 1997, grades 7-10)		
2.2) A variable can be used to represent a fixed but unknown quantity.	• Understand how to use variables to stand for unknowns in equations • Understand how to use variables to model arithmetic story problems containing an unknown (not repeated and repeated)	• Equations or expression using variables to model an arithmetic situation			
2.3) A variable can be used to represent a generalized number in an arithmetic relationship.	• Interpret a variable appropriately as depicting a generalized pattern • Use variables to express arithmetic generalizations	• Mathematical statements (expressions and equations) written using variables to depict generalized patterns			
2.4) A variable can be used to represent a varying quantity.	• Understand that the role of variable in a functional relationship is that of varying quantity • Interpret a variable appropriately as a varying quantity • Use variables to depict varying quantities	• Mathematical statements (expressions or equations) that use variables to depict varying quantities	• Students have difficulty understanding that the value of the quantity can vary		• Which is larger? (Knuth et al., 2005; Weinberg et al., 2004; grades 6-8)

2.5) A variable can be used to represent a parameter whose value determines the characteristics or behavior of other quantities. While the value of a parameter can vary, its value is always fixed in the context of a specific problem.	• Interpret a variable appropriately as a parameter • Use variables to depict a parameter	• Mathematical statements (expressions and equations) that use variables to depict parameters			• Generalized String Problem (adapt from Blanton, 2008)
2.6) The same variable used in the same equation or other context must represent the identical value, while different variables may represent the same value or different values.	• Interpret repeated variables occurring in the same context as representing the same value or values • Interpret distinct variables as able to represent the same or different values • Use variables in appropriate ways to model arithmetic generalizations and situations	• Appropriate use of variables in equations or expressions to model situations requiring repeated unknowns and/or the use of distinct unknowns. • Appropriate solutions of equations such as x + x = 10, illustrating that x = 5 is the solution and that e.g., x = 2 and x = 8 is not an option	• Students sometimes think that, in equations with repeated variables, the variables can represent different quantities (e.g., x + x = 10 might be viewed as having solutions of 2 and 8 for x, since 2 + 8 = 10) (Stacey & MacGregor, 2000, grades 9-10) • Students may not believe that different symbols can have the same value (Carpenter et al., 2003, grades 3-6; Stephens, 2005, grade 6) • Students may not believe that		• Mice in cages (Carpenter et al., 2003, grades 3-6) • Is h + m + n = h + p + n sometimes, always, or never true? (Stephens, 2005, TAAR assessment, grades 6-8) • a = a; c = r (Stephens, 2005; TAAR interviews, grades 6-8)

			equations identical except for the symbol used have the same solution (Wagner, 1977; cited in Herscovics & Kieran, 1999)		
2.7) A variable may represent either a discrete or continuous quantity.	• Understand when a variable represents a discrete quantity or a continuous quantity	• Statements identifying a variable as discrete or continuous			

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