Project LEAP Teacher Professional Development Lessons

Lesson 1
EEEI – Relational Understanding of the Equal Sign

1. Overview of key algebraic thinking practices: Generalizing, representing generalizations, justifying generalizations, and reasoning with generalizations as objects
2. Discuss: How would students solve the problem 8 + 4 = ___ + 5?
3. Teachers solve Classroom Task 1[1] and answer the following:
    a. How might students solve the tasks? What might you do as a teacher (including what materials you might provide) to support them?
    b. What mathematics (algebra, specifically) does each of the items address?
    c. Discuss how tasks can be used for multiple algebraic purposes and how strategic choice of numbers and operations can support algebraic thinking:
        - use of large numbers;
        - use of tasks that simultaneously develop equal sign understanding, introduce fundamental properties and other arithmetic generalizations, and introduce variable as fixed/unknown quantity;
        - algebra tasks can also serve arithmetic purposes (choose number/operation tasks based on your arithmetic objectives, e.g., adding two-digit numbers)
    d. How would you redesign the tasks to fit your grade level? What choices do you make and why?
4. Understandings of the equal sign we want to see in students' thinking:

[1] Depending on the grade level being addressed, the Classroom Task should correlate with the appropriate grade-level intervention. Tasks referenced here are included in Appendix A.

a. Understand that '=' does not mean to just compute numbers to left; evidenced by accurate answers to missing number problems
b. Understand how to reason with quantities to find missing number (rather than just add) – development of relational understanding
c. Understand how to interpret equations/number sentences that don't have a single numerical value on either side.

5. Clip 2.1 (*Thinking Mathematically* – Carpenter, Franke & Levi (2003)*)* – Children's strategies for solving open number sentences; note how she decomposed quantities in 43 + 28 and used Fundamental Properties – this is a critical foundation for developing algebra; note use of increasingly larger numbers; *what happened to help the student develop a relational approach to solving the 5th task?*
6. Clip 1.3 (*Thinking Mathematically* - Carpenter, Franke & Levi (2003)*)* – Why might the teacher have chosen these particular equations? (Note placement of '='); What conceptual issues did students have with these equations? ('=' as signal to compute; reflexive property (Is 6 = 6?)); Note that teacher uses true/false equations to build students' understanding of equality and being able to correctly answer 8 + 4 = ___ + 5)
7. **Developing relational understanding is a process that happens over time – not in one lesson!**

Homework #1:

(1) Implement the Classroom Lesson on Equivalence and write a reflection. Be prepared to discuss and share students' thinking.

(2) What are you teaching? Bring a description of the topics you are currently teaching to share with teachers. We will use these to begin thinking about how we might strengthen these topics or concepts to address the four algebraic thinking practices.

Lesson 2
GA – Fundamental Properties (Additive Identity, Additive Inverse, and Commutative Property of Addition)

(1) Discuss **Relational Understanding of Equality** task; How did students solve the tasks? Did they use arithmetic or algebraic strategies? What surprised you about their thinking? How did you introduce the task? How might you follow-up on what you saw?

(2) **REVIEW:** Understandings of the equal sign we want to see in students' thinking:

- Understand that '=' does not mean to just compute numbers to left of the symbol; evidenced by accurate answers to missing number problems
- Understand how to reason with quantities to find missing number (rather than just compute)
- Understand how to interpret equations/number sentences that don't have a single numerical value on either side.

(3) Discuss Fundamental Properties, why they are important, and what we do with them in Early Algebra (both develop deeper understanding of arithmetic and provide opportunity to engage in the four algebraic thinking practices (generalize, represent generality, etc); Discuss nature of axioms – that we don't "prove" Fundamental Properties are true, they are assumed to be true. But we do want children to convince themselves that the FPs are reasonable and and understand how they operate.

(4) Teachers solve Classroom Task 2 for Fundamental Properties and answer the following:

- How might students solve the tasks?
- What mathematics (algebra, specifically) does each of the items address?
- Discuss how tasks can be used for multiple algebraic purposes and how strategic choice of numbers and operations can support algebraic thinking
- How would you redesign the tasks to fit your grade level?

(5) How do we support children's transition from the use of natural language to variable notation in representing arithmetic generalizations such as Fundamental Properties?

(6) Discuss the topics/concepts teachers will be teaching in the next couple of weeks? Where can you find opportunities to build a relational understanding of equality, or generalize, represent, justify, and reason with the Fundamental Properties?

Homework #2:

1. Implement the Classroom Lesson on Fundamental Properties (Classroom Task 2) and write a reflection. Be prepared to discuss and share students' thinking.
2. Where can you incorporate the ideas of this lesson in grade-appropriate ways in your own teaching? Develop a task involving the Fundamental Properties and algebraic thinking appropriate to your grade. Be prepared to share and discuss your task.

Lesson 3
GA – Evens/Odds and other arithmetic generalizations

1. Discuss teachers' findings from classroom implementation of **Fundamental Properties (Additive Identity, Additive Inverse, and Commutative for Addition)** task;
   - How did students solve the tasks? What notation was useful?
   - Were they able to express the generalizations in words? In variables?
   - What kinds of issues did they have with variables?
   - Did they understand that the properties hold for all values of the variables?
   - Did they understand the reason for using different variables vs repeated variables?
   - What kinds of arguments did they give to support their conjectures (e.g., contrast empirical arguments with more sophisticated arguments such as representation-based arguments)
2. Note: The use of this lesson is somewhat artificial because we are using this at discrete a point in time. FPs should be integrated in a natural way throughout instruction:
   - whenever you look at operations on numbers and
   - when you look at these properties on *different* (extended) domains of numbers
   - when you look at computations (including decomposing numbers) and how building on students' intuitive reasoning helps them look for efficient strategies (as opposed to rote relying on standard algorithms)

Fundamental Properties vs Standard Algorithms:

How would you (or your students) solve: 749 + 31; 378 + 794 = 778 + ____

Sequence of Learning to promote: (1) Use properties implicitly to reason about computation; (2) Explicitly identify properties and describe in natural language; (3) symbolize properties and explore why they are true

3. Have teachers look at concepts they're teaching and tasks ideas they came up with to see where/how they can integrate relational understanding of the equal sign and Fundamental Properties into their daily practice. (e.g., computations where you can lift out fundamental properties; number sentences that can be written in non-standard format; open number sentences that can reveal/address misconceptions about equality)
4. Teachers solve the Classroom Task for Evens and Odds (Classroom Task 3) and answer the following:
   - How might students solve the tasks?

- What mathematics (algebra, specifically) does each of the items address?
- Discuss how tasks can be used for multiple algebraic purposes and arithmetic purposes and how strategic choice of numbers and operations can support algebraic thinking
- How would you redesign the tasks to fit your grade level?

Homework #3:

1. Implement the Classroom Lesson on Evens and Odds (Classroom Task 3) and write a reflection. Be prepared to discuss.

Lesson 4

EEEI (Candy Problem & Extensions): Writing algebraic expressions to model problem situations

1. **Review Relational Understanding of Equal Sign and Arithmetic Generalizations (below)**

2. Discuss what teachers found from **Evens and Odds** task

- How did you introduce the task?
- How did students solve the tasks? What notation was useful?
- Were they able to express the generalizations in words? In variables?
- Did they understand that the conjecture holds for any numbers in the specific class?
- What kinds of **arguments** did they give to support their conjectures (contrast empirical arguments with more sophisticated arguments such as representation-based arguments)

1. Watch Susie video (from *Thinking Mathematically* – Carpenter, Franke & Levi (2003))
   a. Describe her thinking that was arithmetic; describe her thinking that was algebraic.
   b. What kind of **argument** did she build? (Talk about empirical vs representation-based and general arguments). How did her algebraic thinking support her arithmetic thinking and vice versa?

   Examples of using generalizations as objects in a justification:

   i. a + b – b = a in Angela's classes: students reasoned that b – b is 0, so a + 0 must be a.
   ii. students reasoning that the sum of 3 odds is odd because we know that odd + odd is even, so even + odd is odd.

RECALL 4 algebraic thinking practices:

1. generalizing
2. representing generalizations (e.g., words, symbols, graphs, tables)
3. justifying generalizations
4. reasoning with generalizations as objects

2. Teachers solve the **Classroom Task 4** and answer the following:

- How might students solve the tasks? What challenges will they have and how will you address them?
- What mathematics (algebra, specifically) does each of the items address?
- How would you redesign the tasks to fit your grade level (especially for lower grades)?

Things to make sure you stress with this task!!:

- **OBJECT/QUANTITY CONFUSION:** Make sure students understand that the variable represents the quantity (e.g., the number of pieces of candy (or number of trucks) a person has), not the object (e.g., candy).
- **Students will want to assign a numerical value to the number of items a person has.** How do we help move them towards understanding that if we have a quantity whose value we do not know, then we use a variable to represent the unknown amount?

3. (Optional) Making sense of FACT FAMILIES: How can we emphasize the algebraic relationships inherent in understanding why two facts are in the same family? Do students memorize that 5 + 3 = 8 is the same as 8 – 3 = 5? Or do they understand why these equations are equivalent and can they articulate these relationships?

For example, if they have a relational understanding of equality and an intuitive understanding of the Symmetric Property of Equality, they should be able to reason implicitly as follows:

5 + 3 = 8 so (by symmetric property)

8 = 5 + 3

8 – 3 = 5 + 3 – 3 (because need to maintain balance)

8 – 3 = 5

REVIEW so far:

1. **Relational understanding of equality**

a. WHAT YOU CAN DO:

   i. give true/false and open number sentences;

   ii. write number sentences in non-standard formats

   iii. watch out for curricula/tasks that teach students formats such as part + part = whole!!

2. **Arithmetic generalizations**

a. Fundamental Properties (FPs)

   i. using computational work to help students notice, generalize, and express FPs.

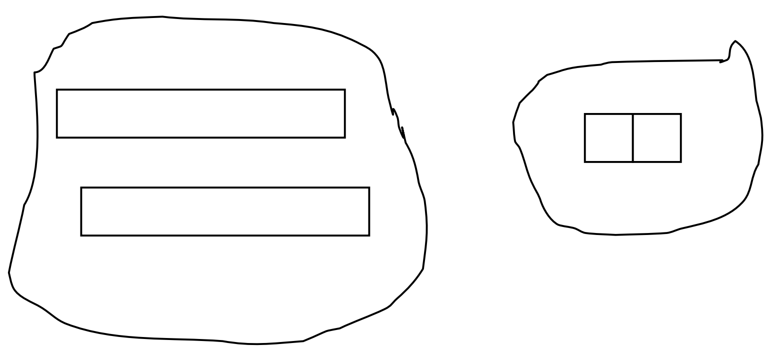
   ii. Getting students to decompose quantities in order to implicitly use FPs

   iii. Getting students to notice where and how they are using FPs in decomposing quantities and be explicit about modeling their words with mathematical statements

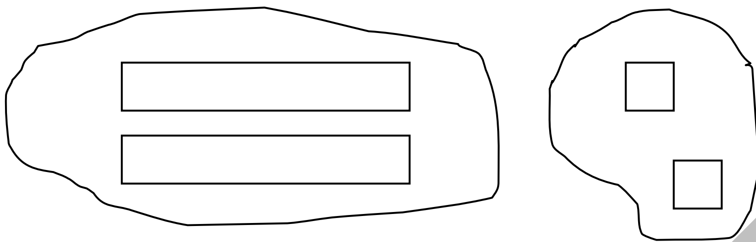
      1. Example: 3rd grader wanted to break down 22 unifix cubes not as two groups of 11 (she said 'it would take way too long to count these out one-by-one), but as two groups of 10, then she "would have just two left over and she could put one with each group".

      So, teacher could call the class's attention to this and model the physical act of how child separated the unifix cubes:

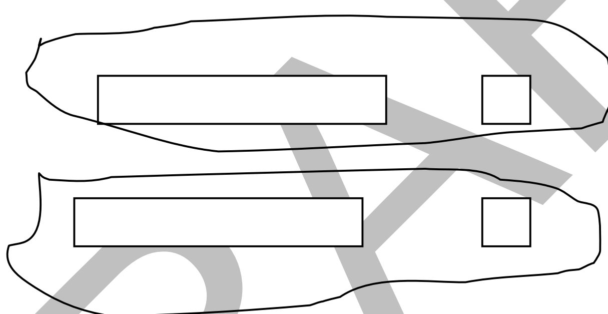
      22 = 10 + 10 + 2



22 = 10 + 10 + 1 + 1



22 = 10 + 1 + 10 + 1 (how can you do this – comm. Prop)



Any time you are operating on numbers – which is the heart of arithmetic!! – there are opportunities for thinking algebraically about the fundamental properties.

b. Other areas where we can make arithmetic generalizations
    i. Classes of numbers (evens/odds)
    ii. Generalizations about factor/divisibility rules
    iii. Is –x always a negative number? Does multiplication always make bigger? Does division always make smaller?

Teacher Homework #4:

1. Implement the Classroom Lesson on the Candy Problem/Truck Problem and write a reflection. Be prepared to discuss.

Lesson 5

EEEI (Candy Problem & Extensions): Writing algebraic expressions to model problem situations; Writing linear equations in one variable to model problem situations; Solving linear equations

1. Discuss student work on Candy Problem and Truck Problem

- How did you introduce the task?
- How did students solve the tasks? What notation was useful? What did they find easy or difficult?
- Were they able to represent the generalizations in words? In variables?
- How did they do on potential challenges?
    - **a.** Do you think they understood what the variable represented (**object/quantity**)
    - **b.** Did they want to assign numerical values to the unknown quantity? How did you handle it?

2. **Look at 'arithmetic' version of Candy Problem.**
Jack and Ava each have a box of candies. They each have 5 pieces of candy in their box. Ava has 4 additional pieces of candy in her hand. How many pieces of candy does Ava have altogether?

3. **Algebrafying a word problem:** Work in groups of three to transform the arithmetic word problem you brought with you into an algebra word problem. Solve the problem. **Share Onion Skin Cells** (Blanton, 2008)

4. Last week, we looked at writing algebraic expressions of the form x + a. We want to extend this work to (1) **modeling equations** and (2) **solving equations***.

***By solving equations – we are NOT talking about solving equations the way one might learn in a high school algebra class (i.e., applying a formal set of procedures), but using understanding of arithmetic and relational understanding of equality to make sense of equations.**

- Clarifying terminology: number sentences vs equations
- Clarifying terminology: algebraic expressions vs equations (with an only-arithmetic background, students try to 'solve' expressions like x + 3)

- **Solve Candy Problem 3-3 & 3-4**
- Note that role of variable takes on two different forms in these tasks: from unknown, varying to unknown, fixed

5. Teachers solve **Classroom Task 5** and answer the following:

- How might students solve the tasks?
- What mathematics (algebra, specifically) does each of the items address?
- Discuss how tasks can be used for multiple algebraic purposes and arithmetic purposes and how strategic choice of numbers and operations can support algebraic thinking
- How would you redesign the tasks to fit your grade level?

Teacher Homework #5:

1. Implement the Classroom Lesson for Tasks 3-3 and 3-4 (review Task 3-2) (**Classroom Task 5**). Give the Review (in groups or individually) and have students discuss their thinking.

Onion Skin Cells

We recently looked at onion skin cells under a microscope. All living things are made of cells. We observed that cells look like boxes and each box has a nucleus. Some pieces of onions had a lot of cells. Others had less. Consider the following information about Onions A, B, and C:

Onion A has an unknown amount of cells. Onion B has 9 more cells than Onion A. Onion C has 4 less cells than onion A.

How would you describe the amount of cells each onion (A, B, C) has? Express your answer any way you can, using an inequality, a number sentence, a word sentence, pictures, tables, or charts.

(from *Algebra and the Elementary Classroom: Transforming Thinking, Transforming Practice* (Blanton, 2008))

Lesson 6
EEEI (Candy Problem & Extensions): Writing algebraic expressions to model problem situations; Writing linear equations in one variable to model problem situations; Solving linear equations

1. Discuss student work on Candy Problem Extensions (Classroom Task 5 from last lesson)

- How did you introduce the task?
- How did students solve the tasks? What notation was useful? What did they find easy or difficult?
- Were they able to express the generalizations in words? In variables?
- How did they do on potential challenges?
    - **a.** Do you think they understood what the variable represented (**object/quantity**)
    - **b.** Did they have difficulty modeling the equation?
    - **c.** Do they understand difference between an expression and equation?
- How did students solve the equation?
- How does this task support their arithmetic understanding?

4. Design your own algebra task. Your task should address at least one of the algebra ideas we've discussed so far:
- Relational understanding of equals sign
- Developing arithmetic generalizations (e.g., Fundamental Properties, generalizations about classes of numbers, factor/divisibility rules, etc.)
- Writing algebraic expressions and equations to model problem situations
- Solving equations by intuitive understanding of equivalent relationships between quantities

5. Solve Classroom Task 6 and answer the following:

- How might students solve the tasks?
- What mathematics (algebra, specifically) does each of the items address?
- Discuss how tasks can be used for multiple algebraic purposes and arithmetic purposes and how strategic choice of numbers and operations can support algebraic thinking
- How would you redesign the tasks to fit your grade level?

Teacher Homework #6:

Implement Classroom Task 6 and write a reflection. Be prepared to discuss.

Lesson 7

Fundamental Properties Revisited

1. Discuss student work on Candy Problem Extensions

- How did you introduce the task?
- How did students solve the tasks? What notation was useful? What did they find easy or difficult?
- Were they able to represent the generalizations in words? In variables?
- How did students think regarding potential challenges or misconceptions?
    - Do you think they understood what the variable represented (**object/quantity**)
    - Did they want to assign numerical values to the unknown quantity? How did you handle it?

2. Review the four algebraic thinking practices in the context of generalizing arithmetic with the fundamental properties
3. Discuss connections between Common Core Mathematical Practices and four algebraic thinking practices
4. Solve and discuss **Classroom Task 7**. Think about the following:
    - How might students solve the tasks?
    - What mathematics (algebra, specifically) does each of the items address?
    - Discuss how tasks can be used for multiple algebraic purposes and arithmetic purposes and how strategic choice of numbers and operations can support algebraic thinking
    - How would you redesign the tasks to fit your grade level?

Teacher Homework #7:
Implement Classroom Task 7 and write a reflection. Be prepared to discuss.

Lesson 8
Functional Thinking

1. Discuss student work on Fundamental Properties (Classroom Task 7)
   - How did you introduce the task?
   - How did students solve the tasks? What notation was useful? What did they find easy or difficult?
   - Were they able to represent the generalizations in words? In variables?
   - How did students think regarding potential challenges or misconceptions?
     - Do you think they understood what the variable represented (**object/quantity**)
     - Did they want to assign numerical values to the unknown quantity? How did you handle it?
- How did students' thinking about FPs differ on these tasks than when it was first introduced?

REVIEW:

2. **Writing algebraic expressions using variables** (e.g., Candy Problem)
   a. object/quantity confusion – understanding what a variable represents

3. **Developing equations to model problem situations; solving equations by using arithmetic strategies**
   a. number sentences vs equations
   b. expressions vs equations

4. **RECALL 4 types of thinking that are central to algebraic thinking:**
   1. generalizing
   2. expressing generalizations (e.g., words, symbols)
   3. justifying generalizations
   4. reasoning with generalizations as objects

All the previous work (relational understanding of equality, expressions and equations, understanding of fundamental properties, and experiences with four essential algebraic thinking practices (generalizing, etc)) will be brought to bear on the development of students' understanding of functions.

2. Teachers solve **Classroom Task 8** and answer the following:

- How might students solve the tasks?
- What mathematics (algebra, specifically) does each of the items address?
- Discuss how function tasks can be used for multiple algebraic purposes and arithmetic purposes and how strategic choice of numbers and operations can support algebraic thinking
- How would you modify this task for your students?
- What will be easy for students? Difficult?
- What do you think students will focus on in this ‘new’ context?

Teacher Homework #8:

Implement Classroom Task 8 and write a reflection. Be prepared to discuss.

Lesson 9
Functional Thinking (Solving Linear Problem Situations for functions of the form y = mx)

1. Discuss student work on Classroom Task 8
   - How did you introduce the task?
   - How did students solve the tasks? What notation was useful? What did they find easy or difficult?
   - Were they able to represent the generalization in words? In variables?
   - How did students think regarding potential challenges or misconceptions?
     - Do you think they understood what the variable represented (**object/quantity**)
     - What kind of relationships did they notice (e.g., recursive, correspondence)

5. Solve **Classroom Task 9 (Trapezoid Problem)** and think about the following:

- What will be easy for students? Difficult?
- What do you think students will focus on in this 'new' context?
- *Discuss leaving values in non-executed form; writing equations (number sentences) to show the relationship between pairs of values*
- *Use problem context to justify the rule*
- *Look at graph/table and think about linear functions and what makes a function linear*

6. Graphing: Conceptual issues students have with Cartesian graphs:

- Think the window defines the extent of the graph (rather than it extending beyond the window
- Units/scale on axes
- Graphing discrete vs continuous quantities (line vs points)

Teacher Homework #9:

Implement Classroom Task 9 and write a reflection. Be prepared to discuss.

Lesson 10
Functional Thinking (Solving Linear Problem Situations for functions of the form y = mx)

1. Discuss student work on Classroom Task 9 (Trapezoid Problem)
   - How did you introduce the task?
   - How did students solve the tasks? What notation was useful? What did they find easy or difficult?
   - Were they able to represent the generalization in words? In variables?
   - How did students think regarding potential challenges or misconceptions?
     - Do you think they understood what the variable represented (**object/quantity**)
     - What kind of relationships did they notice (e.g., recursive, correspondence)

5. Solve **Classroom Task 10 (Outfit Problem)** and think about the following:

- What will be easy for students? Difficult?
- What do you think students will focus on in this 'new' context?
- *Discuss leaving values in non-executed form; writing equations (number sentences) to show the relationship between pairs of values*
- *Use problem context to justify the rule*
- *Look at graph/table and think about linear functions and what makes a function linear*

6. Graphing: Conceptual issues students have with Cartesian graphs:

- Think the window defines the extent of the graph (rather than it extending beyond the window
- Units/scale on axes
- Graphing discrete vs continuous quantities (line vs points)

Teacher Homework #10:

Implement Classroom Task 10 and write a reflection. Be prepared to discuss.

Lesson 11

Functional Thinking (Solving Linear Problem Situations for functions of the form y = x + b)

1. Discuss results of **Classroom Task 10 (**Outfit Problem)

- How did you introduce the task?
- How did students solve the tasks? What notation was useful? What did they find easy or difficult?
- How did they organize the data?
- What types of patterns did they notice (e.g., recursive vs correspondence)
- Were they able to express the function rule in words? In variables?
- What meanings did they give to the variables?

2. Conceptual issues students have with Cartesian graphs:

- Think the window defines the extent of the graph (rather than it extending beyond the window
- Units/scale on axes
- Graphing discrete vs continuous quantities (line vs points)

3. Solve **Classroom Task 11 (Saving for a Bicycle Problem)** and think about the following:

- What will be easy for students? Difficult?
- What do you think students will focus on in this context?
- *Discuss leaving values in non-executed form; writing equations (number sentences) to show the relationship between pairs of values*
- *Use problem context to justify the rule*
- *Look at graph/table and think about linear functions and what makes a function linear*

Teacher Homework #11:

Implement Classroom Task 11 and write a reflection. Be prepared to discuss.

Lesson 12

Functional Thinking (Solving Linear Problem Situations for functions of the form y = mx + b)

1. Discuss results of **Classroom Task 11 (**Saving for a Bicycle Problem)

- How did you introduce the task?
- How did students solve the tasks? What notation was useful? What did they find easy or difficult?
- How did they organize the data?
- What types of patterns did they notice (e.g., recursive vs correspondence)
- Were they able to express the function rule in words? In variables?
- What meanings did they give to the variables?

2. Conceptual issues students have with Cartesian graphs:

- Think the window defines the extent of the graph (rather than it extending beyond the window
- Units/scale on axes
- Graphing discrete vs continuous quantities (line vs points)

3. Solve **Classroom Task 12 (The String Problem)** and think about the following:

- What will be easy for students? Difficult?
- What do you think students will focus on in this context?
- *Discuss leaving values in non-executed form; writing equations (number sentences) to show the relationship between pairs of values*
- *Use problem context to justify the rule*
- *Look at graph/table and think about linear functions and what makes a function linear*

Teacher Homework #12:

Implement Classroom Task 12 and write a reflection. Be prepared to discuss.

APPENDIX A
CLASSROOM TASKS (GRADE 3)

Classroom Task 1: Relational Understanding of Equal Sign

Week 1: Relational Understanding of Equal Sign

Objective:

- Develop a relational understanding of the equal sign by interpreting equations written in various formats (other than a+b=c) as true or false

Jump Start: How would you describe what the symbol '=' means?

EEEI-3-1: Understanding '='

A. Which of the following equations are true? Explain.

- 4 + 6 = 10
- 4 + 6 = 10 + 0
- 10 = 4 + 6
- 10 = 10
- 4 + 6 = 0 + 10
- 2 + 3 = 5 + 4
- 2 + 3 = 1 + 4
- 4 + 6 = 10 + 2
- 4 + 6 = 4 + 6
- 4 + 6 = 6 + 4

(Use student responses to A. to develop the notion that the symbol '=' indicates two quantities are the same.)

C. Write your own true or false equations

D. Introduce the LEAP Math Journal, and have students take 5 minutes to describe what they learned: *Tell me in your own words what the equal sign means. You may use numbers, pictures or words in your definition. Write two true equations.*

Review:

True or False: 12 + 8 = 20 + 5

True or False: 34 = 20 + 14

(Week 2: Relational Understanding of the Equal Sign)

Objective:

- Develop a relational understanding of the equal sign by solving missing value problems. Solutions may be obtained by reasoning from the structural relationship in the equation (compensation strategy) or using arithmetic strategies.

Jump Start:

- Are the following equations true or false? Explain.

  12 + 8 = 20 + 5
  34 = 20 + 14
  5 = 5

EEEI-3-1: Understanding '='

A. What numbers will make the following equations true?

- 4 + 6 = ___ + 6
- 4 + 7 = ___ + 8
- 28 + 3 = ___ + 2
- 28 + 15 = ___ + 14
- 9 + ___ = 8 + 4
- 8 = ___
- 0 + ___ = 21

Discuss strategies students used. Develop compensation strategy.

B. LEAP Math Journal: Show in words, numbers, or pictures how you would find the missing value in the equation 15 + 20 = 14 + _____.

Classroom Task 2: Fundamental Properties

<u>**Week 3: Developing Fundamental Properties**</u>
(Additive Identity & Additive Inverse)

Objective 1: Identify fundamental properties by observing structure in computational work, describe these properties in words and variables, and understand for what values they hold true.

Objective 2: Understand the meaning for using repeated variables to express fundamental properties.

Objective 3: Understand how identify fundamental properties used in computational work and to compute efficiently by using fundamental properties to decompose quantities.

Jump Start: Are these equations true or false? Explain.

8=8+0
0=37-37
23+17=17+23
35+(5+10)=(35+5)+10

GA-3-1: Additive Identity

A. Find the missing numbers:

3 + 0 = ___
____ = 3 + 0
0 + 3 = ___

15 + 0 = ___
___ = 0 + 15
___ = 15 + 0

___ = 0 + 23
23 + 0 = ___
0 + 23 = ___

398 + 0 = ____
0 + 398 = ____
____ = 398 + 0

B. What do you notice? What can you say about what happens when you add zero to a number? Describe your conjecture in words.

C. Represent your conjecture using a variable. Why did you use the same variable? What does it mean to use the same variable in an equation?

D. Can you express your conjecture a different way using the same variable and number?

E. For what numbers is your conjecture true? Is it true for all numbers? Use numbers, pictures, or words to explain your thinking.

G. **Application:** Jenna has 83 pencils. Her mother gives her some more pencils. The next day, she gives her friend Mark the pencils her mother gave her. How many pencils does Jenna have now? Write an equation that represents this situation.

Discuss how this problem uses the Additive Identity property.

Review:

Is 8 = 8 + 0 true or false?

What are the different ways you can write this, using only these numbers, so that the equation is still true?

___ + 0 = ____. What numbers will make this equation true?

GA-3-2: Additive Inverse

A. Find the missing numbers:

0 = 35 – ___

___ – 247 = 0

___ = 78 – 78

B. What do you notice? What can you say about what happens when you subtract a number from itself? Describe your conjecture in words.

C. Represent your conjecture using a variable. Why did you use the same variable? What does it mean to use the same variable in an equation?

D. Can you express your conjecture a different way using the same variable and number?

E. For what numbers is your conjecture true? Is it true for all numbers? Use numbers, pictures, or words to explain your thinking.

G. **Application:**

1. Callie's mother has some juice boxes in her pantry. Callie's friends come over to play and her mother gives everyone a juice box. She doesn't have any left. Write an equation that represents this situation.

Discuss how this problem uses the Additive Inverse Property.

2. a) Compute 10 + 47 – 5 without using an algorithm.

b) Marianne said she solved a) in the following way:

I wrote 10 as 5 + 5, so 10 + 47 – 5 = 5 + 5 + 47 – 5 = 5 + 47 + 5 – 5. Since 5 – 5 is just zero, I know that 10 + 47 – 5 = 5 + 47. But 5 is 2 + 3, so I know that 5 + 47 = 2 + 3 + 47. Since 3 + 47 is 50, then 5 + 47 is 2 + 50, or 52. My answer is 52.

Discuss Marianne's strategy and how she used the ideas in this lesson (Additive Identity and Additive Inverse)

Review:

1. Is 0 = 27 – 27 true or false?
2. ___ - ___ = 0. What numbers will make this equation true?

Week 4: Developing Fundamental Properties (Commutative Property of Addition)

Objective 1: Identify fundamental properties by observing structure in computational work, describe these properties in words and variables, and understand for what values they hold true.

Objective 2: Understand what it means to use multiple variables to express fundamental properties.

Objective 3: Understand how identify fundamental properties used in computational work and to compute efficiently by using fundamental properties to decompose quantities.

Jump Start:

1. Which of the following equations are true? Explain.

    14 – 14 = 0
    394 + 0 = 394
    17 + 5 = 23 + 5
    30+(10+19)=(30+10)+19

2. Marta has 6 pieces of candy. Her friend, Sarah, has 9 pieces of candy. How would you represent the relationship between the number of pieces of candy they have? Using the same numbers, can you represent the relationship in a different way?

GA-3-3: Commutative Property of Addition

A. Which of the following number sentences are true? Use numbers, pictures, or words to explain your reasoning.

    17 + 5 = 5 + 17
    20 + 15 = 15 + 20
    148 + 93 = 93 + 148

B. What numbers or values make the following number sentences true?

25 + 10 = ____ + 25
___ + 237 = 237 + 395
38 + ___ = ____ + 38

C. What do you notice? What can you say about the order in which you add two numbers? Describe your conjecture in words.

D. Represent your conjecture using variables. Why did you use different variables? What does it mean to use different variables in an equation?

E. Can you express your conjecture a different way using the same variables?

F. For what numbers is your conjecture true? Is it true for all numbers? Use numbers, pictures, or words to explain your thinking.

G. **Application:**
Compute the following without using an algorithm:

95 + 39 – 39 + 12

68 + 27 + 32 – 27

Discuss how decomposing quantities and the fundamental properties are used to make computation more efficient.

Review:
1. Is 23 + 17 = 17 + 23 true or false? What are the different ways you can write equation, using only these numbers, so that the equation is still true?

1. ___ + 0 = 0 + ____. What numbers will make this equation true?

2. Kara said that you could use any number in (2) and that she could represent "any number" with a variable. She represented this in the following way:

$$0 + b = b + 0$$

Marcus said he agreed, but he wrote c + 0 = 0 + b. Do you agree with how Marcus? Explain.

Classroom Task 3: Adding Evens and Odds

Week 5: Adding Evens and Odds

Objective 1: Develop arithmetic generalizations about sums of evens and odds;

Objective 2: Develop an understanding of representation-based arguments for justifying a conjecture.

Jump Starts:

1. Find the missing value: 14 + ___ = 15 + 6
2. True or False: 34 + 10 = 44 + 9
3. r + 0 = r; What numbers make this equation true?
4. Compute 19 + 52 – 17 without using an algorithm; *Discuss how decomposing quantities and the fundamental properties can be used to make computation more efficient*

GA-3-5: Sums of Evens and Odds (two addends)

"How Many Pairs?"

Use cubes to answer the following questions:

How many pairs of cubes are in the number 6? How many cubes are left over after you've made your pairs?

Use your cubes to complete the following table for the given numbers.

Number	Number of pairs created	Number of cubes left over
3		
4		
5		
6		
7		

What do you notice? What kinds of numbers have no cubes left over after all pairs are made? What kinds of numbers have a cube left over? Write a sentence to describe each of your observations.

(Allow students sufficient time to represent even and odd numbers with tiles or cubes.)

Part 1

A. Jesse is adding two even numbers. Do you think his answer will be an even number or an odd number?

B. Develop a conjecture to describe what you found.

C. Is your conjecture true for any two even numbers you add together? How do you know? Use numbers, pictures, cubes, or words to explain your thinking.

(Explore the different types of arguments students use.)

Week 6: Adding Evens and Odds (continued)

Objective 1: Develop arithmetic generalizations about sums of evens and odds.

Obejctive 2: Develop an understanding of representation-based arguments for justifying a conjecture.

Jump Start:

1. Find the missing value: 23 – ___ = 24 – 6

2. True or False: 55 + 20 = 75 + 9

3. Jacqueline made $28 babysitting. Her sister, Jenna, made $31. How would you represent the relationship between the amount of money they each earned? Using these same amounts, can you represent your relationship in a different way?

4. Simplify a + 5 – a. Explain how you got your answer. *Discuss how the fundamental properties are used here.*

Part 2

D. Jesse's teacher gives him a new task of adding an even number and an odd number. He thinks his answer will be even. Do you agree? Explain your thinking.

E. Develop a conjecture to describe what you think will happen when you add an even number and an odd number.

F. Is your conjecture true for any odd number and even number you add together? How do you know? Use your cubes or a drawing to explain your thinking.

Part 3

G. Jesse's teacher gives him a third task. He has to add two odd numbers. Do you think his answer will be an even number or an odd number?

H. Develop a conjecture to describe what you found.

I. Is your conjecture true for any two odd numbers you add together? How do you know? Use your cubes or a drawing to explain your thinking.

Classroom Task 4: Candy Problem

Week 7: Modeling Problem Situations with Linear Algebraic Expressions

Objective 1: Understand how to represent a quantity in a problem situation as a (linear) algebraic expression using variables;

Objective 2: understand how to interpret variables and algebraic expressions within a problem context;

Objective 3: Understand how to use fundamental properties to represent algebraic expressions in different, equivalent ways

Jump Starts:
1. Find the missing values:
45 – 10 = ____ – 20
0 = 857 – ___

2. True or False?
m + n = n + m (for any numbers m and n). Why?

3. Even or Odd
Jackson is adding 3 odd numbers. Do you think his result will be even or odd? Why?

3. Compute the following without using an algorithm:
   100 – 50
   101 – 51 + 35

Discuss how decomposing quantities and the fundamental properties are used to make computation efficient

EEEI-3-2 (revised): The Candy Problem (Adapted from Carraher, Schliemann & Schwartz, 2008)

(Use boxes and pieces of candy to help children understand)

A. Jack has a box of candies. He's not sure how many pieces of candy are in his box. His mother gives him 4 more pieces. Can you draw a picture to illustrate this situation?

B. Write a mathematical expression to represent the number of pieces of candy Jack now has.

C. What does the variable in your expression represent?

D. Can you represent the number of pieces of candy Jack has in a different way using the same variable and number?

E. **(Extra problems for group work):**

Ben has some trucks in his collection. He is not sure how many he has. How would you represent the number of trucks Ben has? Ben's mom gives him 5 more trucks for his birthday. Write an expression to represent the total number of trucks Ben now has.

What does the variable in your expression represent? Could you represent his number of trucks any other way using the same number and variable?

Simba loves dog treats. His owner, Chris, has an old box of Simba's treats in the closet. He's not sure how many treats are in the box. How would you represent the number of treats in the box? When Chris goes to the grocery store he buys Simba a new box of 24 treats. Write an expression to represent Simba's total number of treats. What does the variable in your expression represent? Could you represent his number of treats any other way using the same number and variable?

Ellie loves rings. She has a jewelry box filled with rings, but she isn't sure how many she has in her box. She gives her friend Tess five of her rings. Write an expression to represent the total number of rings Ellie now has. (Assume she had more than 5.) What does the variable in your expression represent?

Cameron is collecting corks to construct a toy building. He has a container of corks. He's not sure how many he has in the container. How would you represent the number of corks he has? One day, he loses three of his corks. Write an expression to represent the number of corks he now has. What does the variable in your expression represent? Could you represent his number of treats any other way using the same number and variable?

Will and Lulu collect coins. They have some coins in one piggy bank they share. They don't know how many coins they have. How would you represent the number of coins in the piggy bank? When Will and Lulu's Grandpa comes over for a visit he gives each of them 10 more coins. Write an expression to represent their total number of coins. What does the variable in your expression represent? Could you represent his number of treats any other way using the same number and variable?

Classroom Task 5: Modeling Linear Equations and Inequalities

Week 8: Modeling Problem Situations with Linear Equations and Inequalities

Objective 1: Understand how to represent a quantity in a problem situation as a (linear) algebraic expression;

Objective 2: Understand how to interpret variables and algebraic expressions within a problem context;

Objective 3: Understand how to relate two expressions in an equation or inequality.

JUMP STARTS:

1. 50 + 10 = 55 + *b* What value of *b* will make the equation true? How do you know?

2. 28 + ___ = ___ + 28 What numbers would make the equation true?

3. Kevin adds three even numbers together. Is his answer an even or odd number? How do you know? Use cubes or draw a picture to explain your thinking.

4. Jackson has 24 cookies. His cousin, Rosie 28 cookies. How would you represent the relationship between the number of cookies they each have? Using the same numbers, can you represent your relationship in a different way?

EEEI-3-2/3-3: The Candy Problem (Adapted from Carraher, Schliemann & Schwartz, 2008; Blanton, 2008)

A. Jack and Ava each have a box of candies. Their boxes contain the same number of pieces of candy. Ava has 4 additional pieces of candy in her hand. Draw a picture to illustrate this situation.

B. How would you represent the number of pieces of candy Jack has?

How would you represent the number of pieces of candy Ava has? Using the same variable and number, can you represent the number of pieces of candy Ava has in a different way?

C. Who has more candy, Jack or Ava? How can you use your picture to explain your answer?

D. How would you represent the relationship between the number of pieces of candy Jack has and the number of pieces Ava has in a mathematical sentence?

E. Suppose Ava counted her candy and found that she had 16 pieces. How does this new information relate to how you previously represented the number of candies Ava has? Write an equation that represents what you know about the number of pieces of candy Ava has.

Week 9: Solving Problem Situations Involving One-Step Linear Equations (additive)

Objective 1: Understand how to represent a quantity in a problem situation as a (linear) algebraic expression;

Objective 2: Understand how to interpret variables and algebraic expressions within a problem context;

Objective 3: Understand how express two equivalent expressions in an equation.

Objective 4: Understand how to solve a linear (one-step) equation using arithmetic reasoning and attending to the structure of the problem.

Jump Starts:

1. True or False? 20 = 20 Why?
   True or False? 20 = 20 + 4 Why? How could you make the equation true?

2. How would you complete the following equation?: y + y + 4 = ___ + 4 + y

3. Compute the following without using an algorithm: 678 + 12 – 10

Discuss how decomposing quantities and the fundamental properties can be used to make computation more efficient.

4. David is baking cookies for the holidays. He has some cookies baking in the oven right now. He is not sure how many cookies are in his oven. How could you represent David's total number of cookies? Suppose we know that his friend, Sarah, has more cookies than David and that Sarah has 27 cookies. Write a mathematical sentence that represents the relationship between the number of cookies they each have.

EEEI-3-4: Candy Problem Revisited – Solving Prolem Situations involving One-step Linear Equations (additive)

A. Recall the equation you developed in the previous problem representing the number of pieces of candy Ava has ($x + 4 = 16$). What does the variable represent?

B. Find the value of the variable. Explain how you found your answer.

C. Use cubes or draw a picture to convince your partner that your answer is correct.

Classroom Task 6: Solving Linear Equations

Week 10: Solving Problem Situtions involving One-Step Linear Equations (multiplicative)

Objective 1: Understand how to represent a quantity in a problem situation as a (linear) algebraic expression;

Objective 2: understand how to interpret variables and algebraic expressions within a problem context;

Objective 3: Understand how express two equivalent expressions in an equation.

Objective 4: Understand how to solve a linear (one-step) equation using arithmetic reasoning and attending to the structure of the problem.

Jump Starts:

1. True or False?: 3 + 4 – 5 = 3 + 4 – 5   How do you know?
   True or False?: 3 + 4 – 5 = 3 + 4 – 5 + 10   How can you make the equation true?

2. Compute the following without using an algorithm:
   396 + 14
   396 + 24

Discuss how decomposing quantities and the fundamental properties can be used to make computation more efficient.

3. Find the missing value in the following equations:
   5 + 5 + 5 = ___•5
   4 + 4 + 4 = ___•4
   a + a + a = ___•a

EEEI-3-5: Candy Problem Revisited – Solving Problem Situations involving One-step Linear Equations (multiplicative)

A. Suppose Jack's friend Carter has twice as many pieces of candy as Jack has. How would you describe the number of pieces of candy that Carter has?

B. Carter counted her candies and found that she has 17. Do you think she counted correctly? Why?

C. Suppose Carter counts her candy (correctly) and finds that she has 24 pieces. How does this new information relate to how you previously represented the number of candies Carter has? Write an equation that represents what you know about the number of pieces of candy Carter has.

D. Use your equation to find the number of pieces of candy that Jack has. Explain how you found your answer.

E. Use cubes or draw a picture to convince your partner that your answer is correct.

Classroom Task 7: Fundamental Properties revisited

<u>Week 11: Developing Fundamental Properties</u>

(Multiplicative Identity & Multiplying by Zero)

Objective 1: Identify fundamental properties by observing structure in computational work, describe these properties in words and variables, and understand for what values they hold true.

Objective 2: Understand the meaning for using repeated variables to express fundamental properties.

Objective 3: Understand how identify fundamental properties used in computational work and to compute efficiently by using fundamental properties to decompose quantities.

Jump Starts:

1.David has some gumballs. His brother, Eric, has twice as many gumballs as David. Who has more gumballs, David or Eric? Draw a picture to represent this situation. Write a mathematical sentence that shows the relationship between the number of pieces of gumballs they each have. Using your same variable and number, can you represent this relationship in a different way?

2. Find the missing value: 3 x 4 = ___ x 2. Explain how you got your answer

3. Find the missing value in the following equations:

2 = ___ x 2
2 + 2 = ___ x 2
2 + 2 + 2 = ___ x 2

2 + 2 + 2 +…..+ 2 = ___ x 2
(100 2's)

GA-3-8: (Multiplicative Identity: a × 1 = a)

A. Compute: 15 x 1. Draw an array to explain your answer.
Compute: 1 x 25. Draw an array to explain your answer.
Compute: 248 x 1. Draw an array to explain your answer.

B. Find the missing numbers.
19 x 1 = ___
___ = 1 x 19

398 x 1 = ____
____ = 398 x 1

C. What do you notice? What can you say about what happens when you multiply a number by 1? Describe your conjecture in words.

D. Represent your conjecture using a variable. Why did you use the same variable? What does it mean to use the same variable in an equation?

E. Can you express your conjecture a different way using the same variable and number?

F. For what numbers is your conjecture true? Is it true for all numbers? Use pictures such as an array to explain your thinking.

GA-3-8: Multiplying by Zero (a x 0 = 0)

A. Compute: 15 x 0. Draw an array or use cubes to explain your answer.
Compute: 0 x 25. Draw an array or use cubes to explain your answer.
Compute: 248 x 0. Draw an array or use cubes to explain your answer.

B. Find the missing numbers.
19 x 0 = ___
___ = 0 x 19

398 x 0 = ____
____ = 398 x 0

C. What do you notice? What can you say about what happens when you multiply a number by 0? Describe your conjecture in words.

D. Represent your conjecture using a variable. Why did you use the same variable? What does it mean to use the same variable in an equation?

E. Can you express your conjecture a different way using the same variable and number?

F. For what numbers is your conjecture true? Is it true for all numbers? Use pictures such as an array to explain your thinking.

Classroom Task 8: Solving Linear Problem Situations

Week 12: Solving problem situations involving linear functions of the form y = mx

Objective 1: Understand how to generate co-varying data from a problem situation and organize in a function table.

Objective 2: Understand how to identify variables to represent varying quantities and interpret their meaning within the problem context.

Objective 3: Understand how to identify a recursive pattern in words and use to predict near data.

Objective 4: Understand how to identify a covariational relationship and describe in words.

Objective 5: Understand how to identify a function rule and describe in words and variables.

Objective 6: Understand how to justify a function rule using the table of values (that is, substituting values from the function table in the rule).

Objective 7: Understand the meaning of different variables in a function rule.

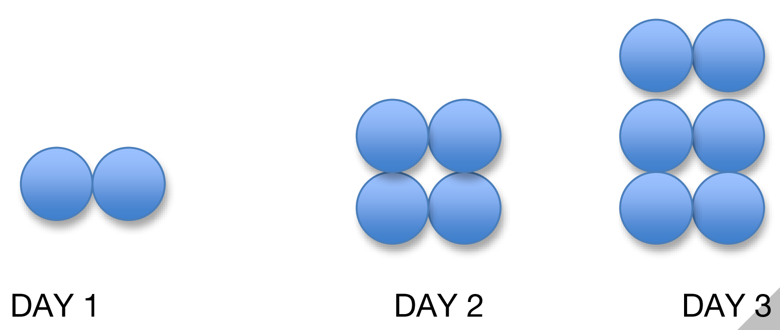
Objective 8: Understand how to reason proportionally about co-varying data to solve problem situations.

Jump Starts:

1. If a < b and b < c, how would you describe the relationship between a and c? Write a story that represents this situation.
2. Find the missing value: 8 x 15 = ___ x 30
3. Find the value of *y* in 3•*y* = 36. Draw a picture to show how you got your answer.

FT-3-4: Growing Circles (adapted from Radford, 2000 “Bingo Chips Pattern”)

A. Each day in math class, Soldenia creates a picture by drawing circles joined together. Following is the picture of circles she drew on each day:



How many circles are in her picture for Day 1? Day 2? Day 3?

What can you say about the number of circles?

B. Organize your information in a table. What two quantities are being compared? How can you represent these in your table?

C. What relationships do you see in the data?

D. Use this to draw what Soldenia's picture might look like on Day 5.

E. Complete the following statement: "As the number of days increases by 1, the number of circles __________".

F. Find a relationship between the day number and the number of circles in the picture on that day. How would you describe your relationship in words?

G. Describe your relationship using variables.

H. Why did you use different variables to represent the two different quantities? Using the same variables and numbers, can you express this rule a different way?

I. Why do you think your relationship is true? How can you use your table to convince your partner that your function rule is true?

J. If the picture on Day 3 needed 6 circles, how many circles would the picture on Day 12 need? Explain how you got your answer.

Classroom Task 9

Weeks 13 & 14: Solving problem situations involving linear function with one operation (multiplicative; y=mx)

Objective 1: Understand how to generate co-varying data from a problem situation and organize in a function table.

Objective 2: Understand how to identify variables to represent varying quantities and interpret their meaning within the problem context.

Objective 3: Understand how to identify a recursive pattern in words and use to predict near data.

Objective 4: Understand how to identify a covariational relationship and describe in words.

Objective 5: Understand how to identify a function rule and describe in words and variables.

Objective 6: Understand how to justify a function rule by reasoning from the context of the problem or using the table of values (that is, substituting values from the function table in the rule).

Objective 7: Understand the meaning of different variables in a function rule.

Objective 8: Understand how to reason proportionally about co-varying data to solve problem situations.

Objective 8: Understand how to construct a coordinate graph, including attending to how discrete data are represented and representing points to scale

Jump Starts:

1. Compute the following without using an algorithm:

23 + 400

23 + 397

14 + 23 + 397 – 10

Discuss how decomposing quantities and the fundamental properties can be used to make computation more efficient.

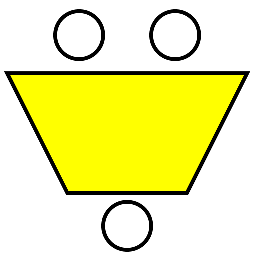
Figure 1

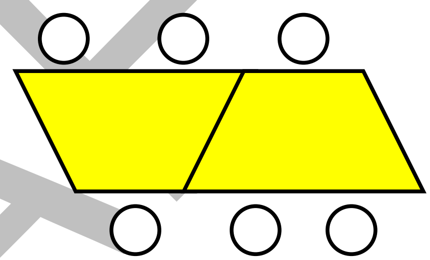
2. Mrs. Gardiner has some pencils to give her students. She gives out 37 of her pencils one Friday afternoon. She counts the remaining pencils and finds that she now has 15. How many pencils did she have to begin with? Use a variable to write an equation that represents this situation.

Figure 2

3-3: Trapezoid Table Problem

A. Suppose you could seat 3 people at a table shaped like a trapezoid (see Fig. 1). If you joined two trapezoid tables end-to-end, you could seat 6 people (see Figure 2). How many people could you seat if you joined three trapezoid tables end-to-end? Four tables?





What can you say about the number of tables?

What can you say about the number of people that can be seated?

B. Organize your information in a table. What two quantities are being compared? How can you represent these in your table?

C. What relationships do you see in the data?

D. Use this to find the number of people that could be seated at 7 tables.

E. Complete the following statement: “As the number of tables increases by 1, the number of people ___________”.

F. Find a relationship between the number of tables and the number of people that can be seated. How would you describe your relationship in words?

G. Use variables to write a rule that describes this relationship.

H. Why did you use different variables to represent the two different quantities?

I. Why do you think your rule is true? How can you use the problem context to justify your rule?

Week 14: (Trapezoid Table Problem continued)

Jump Starts:

1. How could you describe in words what the following equations mean?

   $r + s = s + r$, for any numbers r and s

   $m \times 1 = m$, for any numbers m

2. If $3 \times n = 12$, what is $3 \times n + 2$?

3-3: Trapezoid Table Problem (continued)

A. Recall the rule $b = 3 \bullet a$ from the Trapezoid Table problem, where a represented the number of tables and b represented the number of people that could be seated at those tables. If 3 tables could seat 9 people, how many people could be seated at 10 tables? Explain how you got your answer.

B. How can you use your rule to find how many people could be seated at 48 tables?

C. Use your table to construct a graph that compares the number of tables to the number of people that can be seated.

   Help students think about the scale of the grap, how to represent discrete points on the graph, and how the axes will be labeled.

D. Describe how the graph is growing. Use your graph to predict what the next (unknown) point would be.

Classroom Task 10

Weeks 15 & 16: Solving problem situations involving linear function with one operation (multiplicative; y=mx)

Objective 1: Understand how to generate co-varying data from a problem situation and organize in a function table.

Objective 2: Understand how to identify variables to represent varying quantities and interpret their meaning within the problem context.

Objective 3: Understand how to identify a recursive pattern in words and use to predict near data.

Objective 4: Understand how to identify a covariational relationship and describe in words.

Objective 5: Understand how to identify a function rule and describe in words and variables.

Objective 6: Understand how to justify a function rule by reasoning from the context of the problem or using the table of values (that is, substituting values from the function table in the rule).

Objective 7: Understand the meaning of different variables in a function rule.

Objective 8: Understand how to use the function rule to predice far values.

Objective 9: Understand how to reason proportionally about co-varying data to solve problem situations.

Objective 10: Understand how to construct a coordinate graph, including attending to how discrete data are represented and representing points to scale

Obejctive 11: Understand how to find the value of the independent variable given the value of the dependent variable by "undoing" operations. (reversibility)

Jump Starts:

1. Find the value of *w* in the following equation: 4 x *w* = 20. Draw a picture to show how you got your answer.

2. Marcia was completing the following table in her science class experiment, but her class ended before she could finish it. Can you complete the table for her? Explain your thinking.

number of watermelons	number of seeds
1	10
	18
4	
	42
6	

3-7: The Outfit Problem
Angela needs to buy a uniform for summer camp. Her uniform has to be a pair of shorts and a shirt. She bought 2 pairs of shorts, but still needs to purchase some shirts. She wants to make sure she has enough shirts to wear different outfits throughout the summer.

A. How many outfits could she make if she bought one shirt? What if she bought 2 shirts? What if she bought 3 shirts?Draw a picture to show how you got your answers.
B. What can you say about the number of outfits?
C. Organize your information in a table. What two quantities are you comparing? How would you represent these in your table?
D. What relationships do you see in the data?
E. Use this to find the number of outfits Angela could make if she purchased 8 shirts.
F. Complete the following statement: "As the number of shirts increases by 1, the number of outfits ___________".
G. Find a relationship between the number of shirts and the number of outfits. How would you describe your relationship in words?

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H. Use variables to write a rule that describes this relationship.

I. Why did you use different variables to represent the two different quantities?

J. Why do you think your rule is true? How can you use the problem context to justify your rule?

Week 16: The Outfit Problem (continued)

Jump Starts:

1. Starbucks® has two big jars of coffee beans. We don't know how many beans are in each jar, but we do know that the jar with the decaf coffee has more beans than the jar with the dark roast coffee. Draw a picture that represents this situation. Write a mathematical sentence that represents the relationship between the number of coffee beans in each of the jars. Using the same variables, can you represent your relationship in a different way?

2. What values of n would make the equation $35 = 25 + n$ true? How do you know? Draw a picture to show how you got your answer.

A. Recall the rule $r = 2 \bullet t$ from the Outfit Problem, where t represented the number of shirts and r represented the number of outfits. If 3 shirts gave Angela 6 different outfits, how many different outfits could she get from 12 shirts? Explain how you got your answer.

B. How can you use your rule to find how many different outfits Angela could make with 43 shirts?

C. Use your table to construct a graph that compares the number of shirts with the number of different outfits Angela could make.

Help students think about the scale of the grap, how to represent discrete points on the graph, and how the axes will be labeled.

D. Describe how the graph is growing. Use your graph to predict what the next (unknown) point would be.

E. If Angela needs to have 36 different outfits, how many shirts does she need to buy? Explain how you got your answer.

Classroom Task 11

Week 17: Solving problem situations involving linear function with one operation (additive; y=x + b)

Objective 1: Understand how to generate co-varying data from a problem situation and organize in a function table.

Objective 2: Understand how to identify variables to represent varying quantities and interpret their meaning within the problem context.

Objective 3: Understand how to identify a recursive pattern in words and use to predict near data.

Objective 4: Understand how to identify a covariational relationship and describe in words.

Objective 5: Understand how to identify a function rule and describe in words and variables.

Objective 6: Understand how to justify a function rule by reasoning from the context of the problem or using the table of values (that is, substituting values from the function table in the rule).

Objective 7: Understand the meaning of different variables in a function rule.

Objective 8: Understand how to use the function rule to predice far values.

Objective 9: Understand how to reason proportionally about co-varying data to solve problem situations.

Objective 10: Understand how to construct a coordinate graph, including attending to how discrete data are represented and representing points to scale

Obejctive 11: Understand how to find the value of the independent variable given the value of the dependent variable by "undoing" operations. (reversibility)

Jump Starts:

1. Find the missing value: 34 – ___ = 34 + ____

2. Marcus and Kayla went fishing with their Dad. Kayla caught twice as many fish as Marcus, but we don't know how many Marcus caught. How could you represent the number of fish they each caught? How could you represent the number of fish they caught altogether?

3. Is the following equation true? $a + b - b = 0$ How do you know?

3-1: Saving for a Bicycle

A. Jackson wants to save enough money to buy a bicycle. He starts with $5 in his piggy bank and he gets a $1 allowance at the end of each week. How much money does he have at the end of the first week? The second week? The third week? The fourth week?

B. Organize your information in a table. What two quantities are being compared? How can you represent these in your table?

C. What relationships do you see in the data?

D. Use this to find the amount of money Jackson has after 6 weeks.

E. Complete the following statement: "As the number of weeks increases by 1, the amount of money in Jackson's piggy bank __________".

F. Find a relationship between the number of weeks and the amount of money Jackson has in his piggy bank. How would you describe your relationship in words?

G. Use variables to write a rule that describes this relationship.

H. Why did you use different variables to represent the two different quantities?

I. Why do you think your rule is true? How can you use the problem context to justify your rule?

J. Use your rule to predict the amount of money Jackson will have in his piggy bank after 50 weeks.

K. Use your table to construct a graph that compares the number of weeks to the amount of money Jackson has saved.

   Help students think about the scale of the grap, how to represent discrete points on the graph, and how the axes will be labeled.

L. Describe how your graph is growing. Use your graph to predict what the next (unknown) point would be.

M. If Jackson needs $75 to buy a bike, how many weeks will he need to save? Explain how you got your answer.

cut line

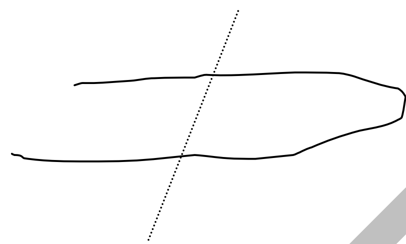
Classroom Task 12

Week 18: Solving problem situations involving linear functions of the form y = mx + b

The String Problem

(adapted from *Mathematics Teaching in the Middle School)*

A. Fold a piece of string to make one loop. While it is folded, make 1 cut (see figure).



How many pieces of string do you have? Fold another piece of string to make one loop. Make 2 cuts and find the number of pieces of string. Repeat this for 3, 4, and 5 cuts.

B. What quantities are you comparing? Use a variable to represent these quantities. Why did you use different letters to represent the two different quantities (*presuming they did*)?

C. Organize your information in a table.

D. What patterns or relationships do you see in the data? Use this to predict the number of pieces of string you would have after 8 cuts.

E. Find a relationship between the number of cuts and the number of pieces of string. How would you describe your relationship in words? Describe this relationship using your variables.

F. Why do you think your relationship is true?

G. If you folded a piece of string and cut it 80 times, how many pieces of string would you get?

H. Suppose your friend had 15 pieces of cut string. How many cuts did your friend make in order to get 15 pieces of string? How did you get your answer?
If your friend said she counted 16 pieces of cut string, what would you say?

I. Construct a graph that shows the number of pieces of string for each number of cuts. How did you represent your data? How did you label the axes? Could more points be represented on your graph? How far could you extend your graph?